Graphs of the Other 4 Trig Functions (3.8+3.11) Homework

1. Find the period of each function.

   a. $y = 4\tan\left(\frac{1}{3}x\right) + 1$

   b. $y = \cot\left(-\pi x + \frac{\pi}{2}\right)$

2. Consider the graph of $y = 3\tan(2x - \pi) + 4$.

   a. All points of inflection for this graph can be found on the line $y = k$. Find k.

   b. Circle all of the following that represent x-values at which the graph of this function has a vertical asymptote.

0	$\frac{2\pi}{3}$	$\frac{3\pi}{2}$	$-\frac{\pi}{4}$	$\frac{\pi}{2}$
$\frac{\pi}{4}$	$\frac{\pi}{8}$	$\frac{5\pi}{4}$	$\frac{\pi}{6}$	$-\pi$

Questions #3-5 pertain to the following set of functions.

$f(x) = -\tan(x)$	$g(x) = \tan(-x)$	$h(x) = \tan(x)$
$j(x) = -\cot(x)$	$k(x) = \cot(-x)$	$m(x) = \cot(x)$

3. Luca picks a function out of the above set and says "I'm thinking of a function. Between consecutive asymptotes, this function increases." State all the functions (from the above set) about which Luca may be thinking.

4. Riley picks a function out of the above set and says "I'm thinking of a function. On the interval $\left(0, \frac{\pi}{2}\right)$, this function's rate of change decreases." State all the functions (from the above set) about which Riley may be thinking.

5. Merida picks a function out of the above set and says "I'm thinking of a function. On the interval $\left(0, \frac{\pi}{2}\right)$, this function is negative." State all the functions (from the above set) about which Merida may be thinking.

6. At which of the following x-values does the graph of $y = \sec\left(3\left(x - \frac{\pi}{6}\right)\right)$ have a vertical asymptote?

$x = -\frac{\pi}{2}$	$x = -\frac{\pi}{6}$	$x = 0$
$x = \frac{\pi}{3}$	$x = \frac{\pi}{2}$	$x = \frac{2\pi}{3}$

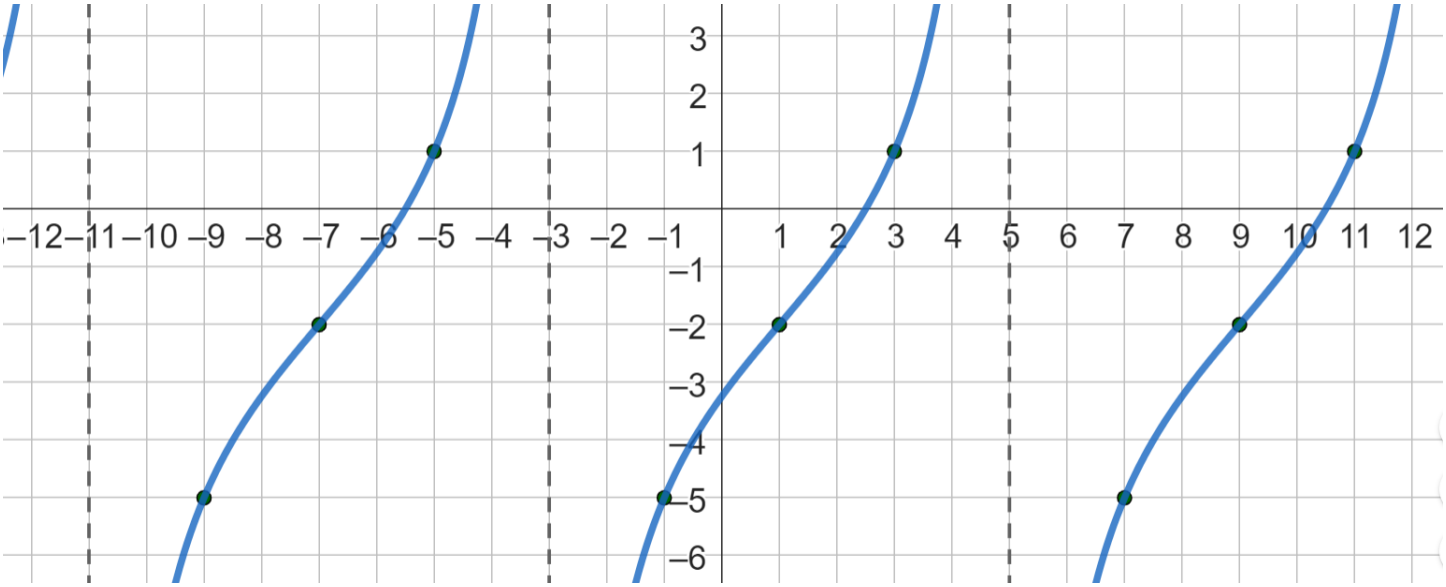
7. Find the range of $f(x) = 3\sec\left(\frac{1}{3}x\right) + 2$. Then, find the equation of four vertical asymptotes for this function.

8. At which of the following x-values does the graph of $y = 3\csc(4x) - 1$ have a vertical asymptote?

$x = -\frac{\pi}{2}$	$x = -\frac{\pi}{6}$	$x = 0$
$x = \frac{\pi}{4}$	$x = \frac{\pi}{2}$	$x = \frac{2\pi}{3}$

9. Find the range of $f(x) = -\frac{1}{2}\csc(\pi x) + 10$. Then, find the equation of four vertical asymptotes for this function.

Below is a graph of $f(x) = a\tan\big(b(x+c)\big) + d$ meant to be used for questions #10-14.



10. Find a
11. Find b
12. Find c
13. Find d
14. Find $\lim_{x\to 5^-} f(x)$

15. Randall is thinking of the graph of the function $f(x) = \tan x$. He performs a vertical dilation by a factor of 3. Then he reflects the graph over the y-axis. Then he translates the graph horizontally by $-\frac{\pi}{4}$ units. The result is the graph of which of the following functions?

    (A) $3\tan\left(-\left(x+\frac{\pi}{4}\right)\right)$

    (B) $-3\tan\left(x-\frac{\pi}{4}\right)$

    (C) $\frac{1}{3}\tan\left(-x+\frac{\pi}{4}\right)$

    (D) $-\frac{1}{3}\tan\left(x-\frac{\pi}{4}\right)$

16. Buzz is thinking about the function $g(x) = \tan(px) + k$, where p and k are constants. The period of $g(x)$ is $\frac{\pi}{4}$. All points of inflection on the graph of $y = g(x)$ lie on the line $y = -3$. Find the values of p and k.

Match each numbered function with the letter of the statement that describes it. There are no repeats.

17. $y = \cot\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 1$
18. $y = \tan\left(\frac{x}{2}\right)$
19. $y = \tan(2x) + 1$
20. $y = \cot(2x)$

A) This odd function decreases between consecutive vertical asymptotes.

B) This odd function has period 2π

C) The asymptotes of this function are all of the form $x = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$

D) This function increases between consecutive vertical asymptotes. All points of its points of inflection can be found on the line $y = 1$.

In questions #21-26, match each numbered function with the letter of the corresponding graph. Use each function/graph once. There are no repeats.

21. $y = 2\tan(2x) + 2$

22. $y = \tan(-2x)$

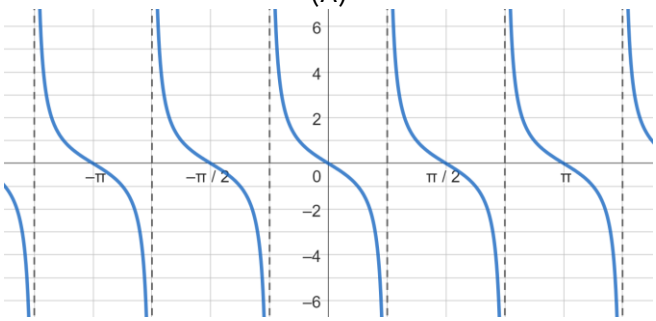
23. $y = 2\cot\left(x + \frac{\pi}{2}\right)$

24. $y = \cot\left(\frac{1}{2}x\right) + 2$

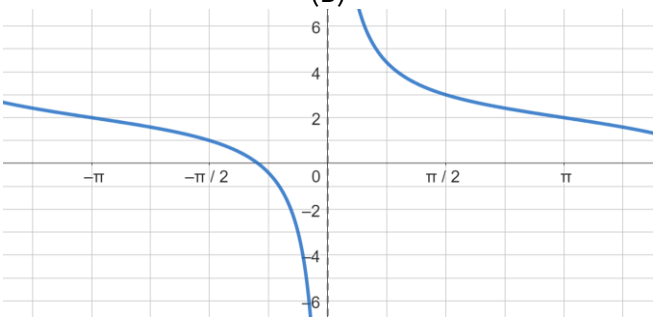
25. $y = 2\tan\left(x - \frac{\pi}{2}\right) + 2$

26. $y = -\tan\left(\frac{1}{2}x\right) + 2$

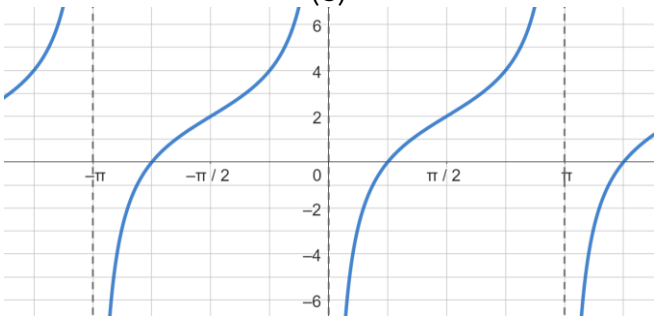
(A)



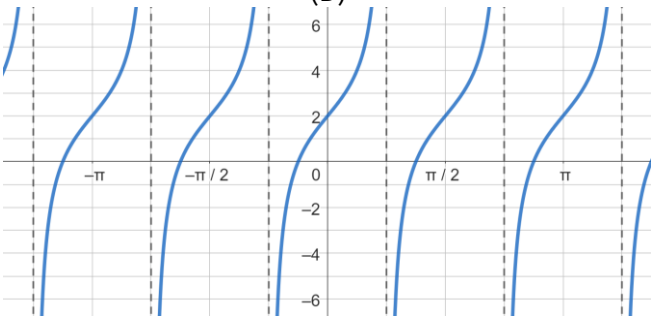
(B)



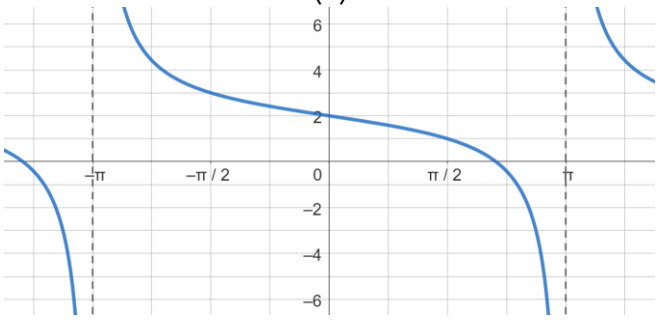
(C)



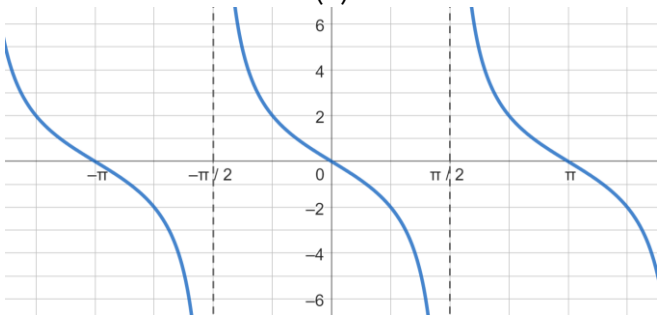
(D)



(E)



(F)



Graphs of the Other 4 Trig Functions (3.8+3.11) HW Solutions

1.
   a. 3π
   b. 1
2.
   a. $k = 4$
   b. $-\frac{\pi}{4}$, $\frac{\pi}{4}$, and $\frac{5\pi}{4}$
3. $h(x) = \tan(x)$, $j(x) = -\cot(x)$, and $k(x) = \cot(-x)$
4. $f(x) = -\tan(x)$, $g(x) = \tan(-x)$, $j(x) = -\cot(x)$, and $k(x) = \cot(-x)$
5. $f(x) = -\tan(x)$, $g(x) = \tan(-x)$, $j(x) = -\cot(x)$, and $k(x) = \cot(-x)$
6. $x = 0$, $x = \frac{\pi}{3}$, and $x = \frac{2\pi}{3}$
7. Range: $(-\infty, -1] \cup [5, \infty)$
   Any equation of the form $x = \frac{3\pi}{2} + 3\pi k$ where k is an integer will be a vertical asymptote of this function.
   So, correct answers include $x = -\frac{9\pi}{2}, -\frac{3\pi}{2}, \frac{3\pi}{2}, \frac{9\pi}{2}, \frac{15\pi}{2}, \ldots$
8. $x = -\frac{\pi}{2}$, $x = 0$, $x = \frac{\pi}{4}$, and $x = \frac{\pi}{2}$
9. Range: $(-\infty, 9.5] \cup [10.5, \infty)$
   Any equation of the form $x = k$ where k is an integer will be a vertical asymptote of this function. So, correct answers include $x = -2, -1, 0, 1, 2, 3, \ldots$
10. $a = 3$
11. $b = \frac{\pi}{8}$
12. There are infinitely many correct answers for c. Any one of the following would be correct.
    $c = -9$ or $c = -1$ or $c = 7$ or $c = 15$ or ...
13. $d = -2$
14. ∞
15. A
16. $p = 4$ and $k = -3$
17. C
18. B
19. D
20. A
21. D
22. A
23. F
24. B
25. C
26. E